5 PLUME RISE

In most cases, pollutants injected into ambient air possess a higher temperature than the surrounding air. Most industrial pollutants, moreover, are emitted from smokestacks or chimneys and therefore possess an initial vertical momentum. Both factors (thermal buoyancy and vertical momentum) contribute to increasing the average height of the plume above that of the smokestack. This process terminates when the plume's initial buoyancy is lost by mixing with ambient air.

The physical consequence of the above phenomenon is generally quantified by a single parameter, the plume rise Δh, defined as the vertical displacement of the plume in this initial dispersion phase. Several studies have provided semiempirical formulae for evaluating Δh (e.g., Briggs, 1975); others have provided more complex and comprehensive descriptions of the several physical interactions between the plume and the ambient air (e.g., Golay, 1982).

5.1 SEMIEMPIRICAL Δh FORMULATIONS

A review of the available semiempirical formulations for computing Δh (and its variation with the downwind distance from the source) is presented by Strom (see Stern, 1976) and Hanna et al. (1982). Many equations for Δh have the following form

$$\Delta h(x) \; = \; const \; Q_h^a \, x^b \, u^c \qquad (5\text{-}1)$$

where a, b, c are constants, x is the downwind distance, and u is the wind speed at z_s, the source height. Q_h is the heat emission rate of the source and is given by

$$Q_h \; = \; Q_m \, c_p \, (T_s - T_a) \qquad (5\text{-}2)$$

where c_p is the specific heat at constant pressure, T_s is the gas exit temperature, T_a is the ambient temperature at z_s, and Q_m is the total mass emission rate given by

$$Q_m = \varrho_s \pi r_s^2 v_s \tag{5–3}$$

where ϱ_s is the density of the total emission, r_s is the exit radius (or the equivalent radius $r_s = \sqrt{A/\pi}$, for a noncircular exit with area A), and v_s is the vertical exit speed.

Among the various schemes, the Briggs (1969) method is one of the most widely known. That method defines the buoyancy flux parameter F_b by

$$F_b = g\, v_s\, r_s^2\, (T_s - T_a)/T_s \tag{5–4}$$

and the critical downwind distance x^* by

$$x^* = 2.16\, F_b^{2/5}\, z_s^{3/5} \tag{5–5}$$

for $z_s < 305$ m, and

$$x^* = 67\, F_b^{2/5} \tag{5–6}$$

for $z_s \geq 305$ m, where z_s is the source height. The critical distance x^* separates the two stages of the plume rise, as discussed below.

For $x \leq x^*$, the plume rise behaves, for all atmospheric stabilities, according to the "2/3 law," i.e., following the formula

$$\Delta h(x) = const\, F_b^{1/3}\, u^{-1}\, x^{2/3} \tag{5–7}$$

with *const* between 1.6 and 1.8, with a suggested value of 1.6 (Briggs, 1972). For $x > x^*$, where ambient atmospheric turbulence plays a dominant role, the plume rise formula becomes, for all atmospheric stabilities,

$$\Delta h(x) = 1.6\, F_b^{1/3}\, u^{-1}\, x^{*\,2/3} \left[\frac{2}{5} + \frac{16\,x}{25\,x^*} + \frac{11}{5}\left(\frac{x}{x^*}\right)^2\right]\left(1 + \frac{4\,x}{5\,x^*}\right)^{-2} \tag{5–8}$$

Subsequently, Briggs (1975) improved the equations for final rise due to turbulence by parameterizing atmospheric turbulence and using appropriate physical quantities (u_*, w_*, heat flux, and height above the ground). He provided separate formulae for mechanically and convectively induced turbulence. This new formulation was used by Turner (1985) as discussed below.

Other formulations have been proposed by Holland (1953), Brummage (1966), Bringfelt (1969), Fay et al. (1970), Carpenter et al. (1971), and many others.

The above equations are used for buoyant plumes; i.e., when $T_s > T_a$. Jets (i.e., nonbuoyant plumes with $T_s \simeq T_a$) can also be treated by similar equations. For example, according to Briggs (1969), the plume rise of a jet is

$$\Delta h(x) = 2.3\, F_m^{1/3}\, u^{-2/3}\, x^{1/3} \tag{5-9}$$

where the momentum flux parameter F_m is

$$F_m = v_s^2\, r_s^2\, \varrho_s/\varrho \simeq v_s^2\, r_s^2 \tag{5-10}$$

where the last equality in Equation 5-10 is valid for emissions with mass density ϱ_s similar to the air density ϱ. The final rise of a jet in stable conditions is given (Briggs, 1975) by

$$\Delta h = 2.6\, (F_b/u\, s)^{1/3} \tag{5-11}$$

where s is the stability parameter defined below.

In calm conditions (i.e., u less than 1 m s^{-1}), the above formulae cannot be used. In these situations, Briggs (1975) suggests the following equations for the final plume rise in stable conditions:

$$\Delta h = 5.0\, F^{1/4}\, s^{-3/8} \tag{5-12}$$

for buoyant plumes, and

$$\Delta h = 4.0\, F_m^{1/4}\, s^{-1/4} \tag{5-13}$$

for jets, where s is the stability parameter

$$s = \frac{g}{\theta} \frac{\partial \theta}{\partial z} \tag{5-14}$$

and θ is the potential temperature.

Briggs's formulae have been incorporated into most of the U.S. EPA models described in Section 14.1. These formulae represent a reasonable compromise between accuracy and simplicity, even though, according to many (e.g., Henderson–Sellers and Allen, 1985), they may tend to overestimate the plume rise at large distances downwind.

Turner (1985) used the Briggs (1975) formulae and proposed a generalized routine that calculates both plume rise and partial penetration of the plume into the layer above the mixing height. This routine assumes that meteorological data (temperature and wind speed) are available by layers and that the mixing height h and the potential temperature gradient $\partial\theta/\partial z$ above the mixing height are known. The method is based on the following computational steps.

1. Calculation of f, the stack tip downwash correction factor, with the method of Bjorklund and Bowers (1982). The parameter f is computed by first evaluating the Froude number F_r

$$F_r = \frac{v_s^2}{2\, g\, r_s\, (T_b - T_a)/T_a} \tag{5-15}$$

Then, if $F_r < 3$, $f = 1$; otherwise ($F_r \geq 3$) we have the following cases:

- if $v_s \leq u$

$$\Delta h = 0 \tag{5-16}$$

and no further plume rise calculations are required

- if $v_s > 1.5\, u$

$$f = 1 \tag{5-17a}$$

- if $u < v_s \leq 1.5\, u$

$$f = 3(v_s - u)/v_s \tag{5-17b}$$

2. Calculation of the final plume rise Δh by layers. If the plume rise exceeds the top of a layer, computations are repeated for the next layer above

using the residual plume buoyancy. Computations are made using formulae similar to those by Briggs described above.

3. Calculation of the actual final plume rise $\Delta h'$

$$\Delta h' = f \Delta h \tag{5-18}$$

to incorporate stack tip downwash effects when $f < 1$.

4. Incorporation of plume penetration of the mixing height by decreasing the emission rate Q and further adjusting the plume rise. More specifically, the bottom b_p and the top t_p of the plume are computed by

$$b_p = z_s + 0.5\,\Delta h' \tag{5-19}$$

and

$$t_p = z_s + 1.5\,\Delta h' \tag{5-20}$$

If the bottom of the plume is higher than the mixing height (i.e., $b_p \geq h$), Q is set equal to 0, since the plume is assumed to make no contribution inside the mixing layer and to remain trapped in the stable layer above it. Otherwise, if $b_p < h$ and $t_p > h$, only a fraction of the plume has remained inside the mixing layer. In this case, the plume rise is further corrected as

$$\Delta h'' = \frac{h - b_p}{2} - z_s \tag{5-21}$$

and the emission rate Q is substituted with an effective rate of $f'Q$, where

$$f' = \frac{h - b_p}{\Delta h'} \tag{5-22}$$

in order to exclude the fraction $(1 - f')$ of the plume which has perforated the mixing height h.

This plume rise/partial penetration technique provides a computationally simple solution for engineering calculations. However, the problem of correctly modeling partial penetration is still wide open. Turner's plume rise routine described above has been incorporated into the AVACTA II package (Zannetti et al., 1986) as a user option and into the Urban Airshed Model described in Section 14.1.1.

5.2 ADVANCED PLUME RISE MODELS

The semiempirical formulations presented in the previous section have shown, on several occasions, a great degree of uncertainty. Additional methods have been proposed that provide, at least in theory, a better physical representation of the two basic phenomena (see Figure 5–1) related to the plume rise:

1. The vertical increase of the plume centerline.
2. The entrainment of ambient air into the plume and its consequent horizontal and vertical spreading.

Briggs (1975) tabulated the characteristics of 22 "basic" plume rise models and many more have been developed since then. It is hopeless to review them all. Brief considerations of some of them are presented below.

The integral plume rise model of Schatzmann (1979) allows a numerical solution of the equations of the conservation of mass, momentum, concentration and thermal energy. This method seems particularly effective (at least close to the source), since it does not use the common Boussinesq approximation and, therefore, allows the treatment of jet flow with density greatly different from that of ambient air. This model, however, fails to account for the inertia of "effective mass" outside the plume, seems to contain an unrealistic drag term, and shows problems in the mass conservation equation (Briggs, personal communication).

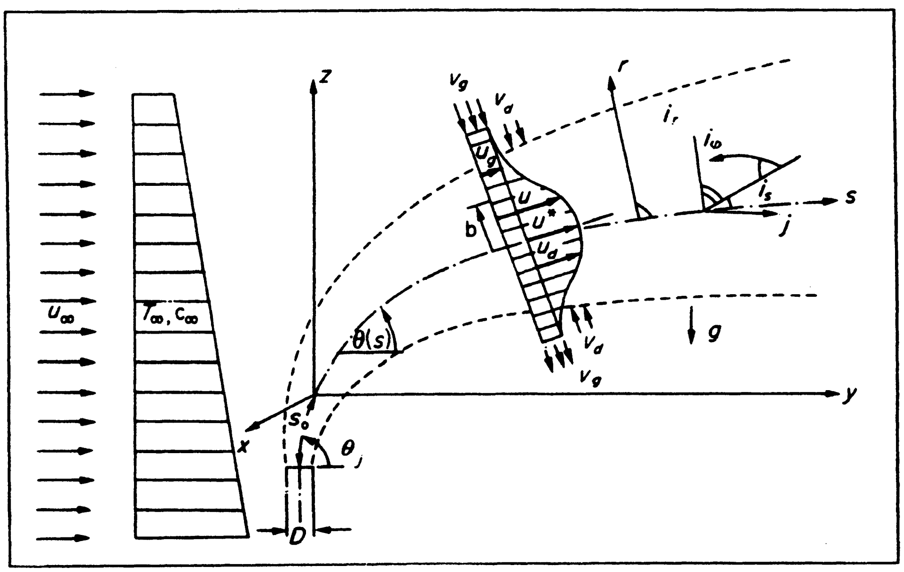

Figure 5–1. Schematic representation of plume rise and entrainment phenomena (from Schatzmann, 1979). [Reprinted with permission from Pergamon Press.]

Golay (1982) has proposed an even more complex approach, a differential entrainment model. It is able to simulate bent-over plumes in complicated vertical atmospheric structures by numerically integrating the conservation equations of mass, momentum, heat, water vapor, liquid water, and the two equations for the turbulent kinetic energy and eddy viscosity in the form presented by Stuhmiller (1974). The major limitation of Golay's approach is the detailed meteorological information that is required; i.e., the vertical profiles of wind speed, virtual potential temperature, relative humidity, turbulence kinetic energy, and turbulent viscosity.

Glendening et al. (1984) have proposed a simpler approach, which numerically integrates the conservation equations, using, however, several simplifying assumptions (the plume is axisymmetric and the three-dimensionality of the plume is ignored).

Henderson-Sellers (1987) has developed a comprehensive model that encompasses both plume rise and pollutant dispersion within a single numerical model formulation. Results are expressed in terms of centerline trajectories, entrainment velocities, rates of spread and ground-level concentrations. The model is also applicable to cases of nonuniform wind and temperature fields as well as to urban terrain. The model has been implemented into an advanced software package (called PRISE) that can run on the IBM PC or compatibles. The code calculates all the phases of the plume (rising, bending over, and [quasi-] equilibrium dispersion) in one continuous formulation.

Probably the most promising technique for the simulation of buoyant plumes in unstable conditions is large eddy simulation (see Section 6.5.2). Nieuwstadt and de Valk (1987) applied such a model to a line source, in which buoyancy was added by increasing the temperature of the source with respect to the ambient temperature. Further work in this direction was performed by van Haren and Nieuwstadt (1989), who obtained reasonable agreement between the output of the large eddy simulation model and the field experiments of Carras and Williams (1984). These large eddy simulation results allow differentiation between the fraction of plume motion caused by convective turbulence and that caused by plume buoyancy. The latter does not seem to obey Briggs' 2/3 law.

Another advanced and computationally-intensive procedure is the Stack Exhaust Model (SEM) developed by Sykes et al. (1989). The SEM model is the most detailed member of a hierarchy of atmospheric models developed for the Electric Power Research Institute (EPRI). This model uses state-of-the-art turbulent simulation techniques in an effort to simulate the initial phase of the plume, including its buoyant rise and bending-over phase. SEM uses the Reynolds-averaged Navier-Stokes equations, under the assumption of an incompressible, Boussinesq fluid. These equations are solved numerically using second-order finite-difference techniques. The model generally provides steady-state solutions, although it is capable of simulating time-dependent flows.

5.3 SPECIAL CASES

Some special plume rise situations have been investigated and, sometimes, special ad-hoc formulae have been provided. Three cases, in particular, need to be mentioned: the plume rise from multiple sources, the partial perforation of an elevated inversion by a plume, and the plume rise from stacks with scrubbers. These cases are discussed briefly below.

5.3.1 Multiple Sources

Briggs (1975) provided a semiempirical formulation for determining the plume rise from several similar stacks close to each other. Anfossi et al. (1978) and Anfossi (1985) developed and tested a virtual stack concept that allows two or more adjacent sources of different heights and emissions to be merged. In general, interactions among adjacent sources produce an enhancement of their plume rise. Multiple source models are also reviewed by Briggs (1984).

5.3.2 Inversion Partial Penetration

Plume buoyancy is often large enough to allow plumes to perforate, or partially penetrate, an elevated temperature inversion layer (see Figure 5-2). The evaluation of this effect is often critical, especially during daytime conditions when a plume below the inversion is easily diffused toward the ground, while a plume above makes little or no concentration contribution in the PBL. Strom (in Stern, 1976) and Turner (1985) provide simple methods for discriminating between these two cases, as discussed at the end of Section 5.1. Manins (1979) investigates a plume's partial penetration in greater detail suggesting, among other things, that a complete plume penetration is almost impossible since, upon reaching the inversion, there will always be a portion of the plume with insufficient buoyancy for further rise. Similar conclusions have also been shown by semi-quantitative buoyant plume simulations by Lagrangian particle methods (Zannetti and Al-Madani, 1983a and 1983b), which allow a high degree of resolution in the representation of the plume and show a typical behavior, in which part of the plume is reflected by an elevated inversion, part is trapped inside it and part is able to perforate it, reaching the layer above.

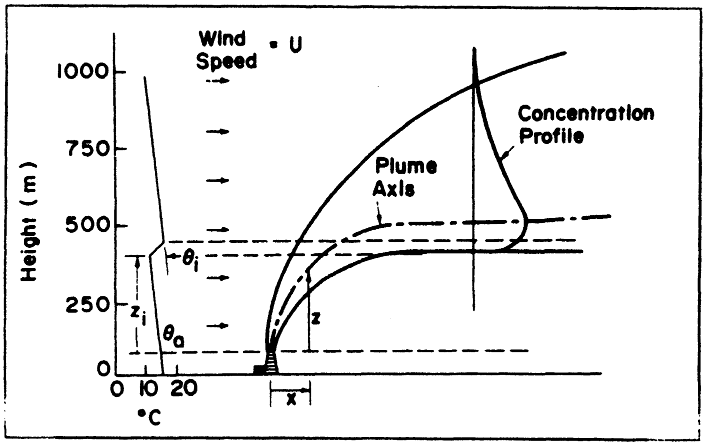

Figure 5–2. Schematic of the interaction of a buoyant plume and an elevated inversion layer (from Manins, 1979). [Reprinted with permission from Permagon Press.]

5.3.3 Stacks with Scrubber

Desulfurization techniques have often been adopted for either the combustibles (e.g., coal cleaning) or the flue gas (scrubbers). The latter technique seems by far the most cost effective for SO_2 emission reduction. Most flue gas desulfurization devices employ a wet scrubbing technique in which a $Ca(OH)_2$ solution is used for partial removal of SO_2.

Plumes from stacks with scrubbers are frequently modeled using the same techniques as the other plumes. Schatzmann and Policastro (1984) reviewed the problem of evaluating Δh for stacks with scrubbers, concluding that "the significant moisture content of the scrubbed plume upon exit leads to important thermodynamic effects during plume rise that are unaccounted for in the usual dry plume rise theories."

Plume rise models for wet plumes (e.g., cooling tower plumes) have been developed by Hanna (1972), Weil (1974) and Wigley and Slawson (1975). Even these formulations, however, are inappropriate for scrubbed plumes, according to Schatzmann and Policastro (1984), because of the simplifications they adopt. Sutherland and Spangler (1980) compared observed plume rise heights for scrubbed and unscrubbed plumes and evaluated the performance of several plume rise formulations. They found that simple plume rise formulae are questionable even for dry plumes, while moisture effects in scrubbed plumes increase the plume buoyancy and almost compensate for the loss of plume rise due to the temperature decrease induced by the scrubbing system. Plume rise of moist plumes has been reviewed by Briggs (1984).

Schatzmann and Policastro (1984) recommend integral-type models for scrubbed plumes, with the additional requirement of avoiding some common simplifications such as the linearization of the equation of state, first-order approximations in the calculation of the local saturation deficit, and the Boussinesq approximation.

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